# Introduction to Programming: Lecture 20

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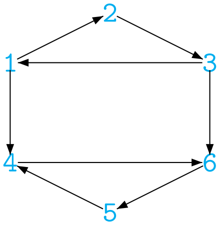
# Graphs



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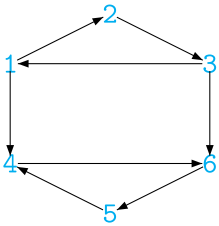

- Represent edges in the graph as a function

```
type Vert = Int
maxvert = 6
edge :: Vert -> Vert -> Bool
edge 1 2  = True
edge 1 4  = True
...
edge 5 6  = True
edge  _   _   = False
```

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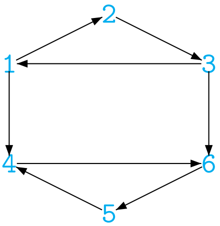

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- Goal: define `reachable ::  Vert -> [Vert]`

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- Inductive definition of `reachable v`
  - `v` is reachable from `v`
  - `x` is reachable from `v` and `edge x y` then `y` is also reachable from `v`.
- Cannot directly translate this definition into Haskell
- `extend` picks up the neighbours of a given vertex.

```
extend :: Vert -> [Vert]
extend v = [w | w <- [1..maxvert], edge v w]
```

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  or remove duplicates after each level is generated.

# Without Duplicates

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where `remDup` removes duplicates in a list.

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  This is the function `step`.

# Avoiding repetition

```
step:: ([Vert], [Vert]) -> ([Vert], [Vert])
step  (s,[])  =  s
step  (s,x:t) = (x:s,nt)
 where
  tmp = extend x
  nt = t ++ filter g tmp
  g y = not (elems y s) && not (elems y t)
```

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- Iterate `step` to get the answer.

```
iterstep (s,t)
  | (t == []) = (s,[])
  | otherwise = iterstep  (step (s,t))

reachable v = iterstep ([],[v])
```

# Search problems

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  - In each new row, place queen at leftmost safe square
- After 7 moves we find no safe squares on bottom row

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Q | | | | | | | |
| | | Q | | | | | |
| | | | | Q | | | |
| | | | | | | Q | |
| | Q | | | | | | |
| | | | Q | | | | |
| | | | | | Q | | |
| | | | | | | | |

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- Similarly go back to 5th queen, 4th queen, . . . , 1st queen

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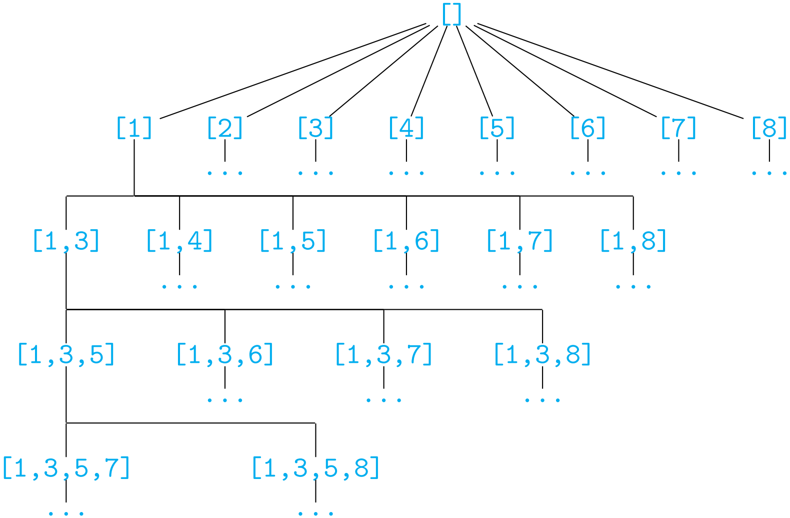

[]

[1] [2] [3] [4] [5] [6] [7] [8]

. . . . . . . . . . . . . . . . . . . . .

[1,3] [1,4] [1,5] [1,6] [1,7] [1,8]

. . . . . . . . . . . . . . .

[1,3,5] [1,3,6] [1,3,7] [1,3,8]

. . . . . . . . .

[1,3,5,7] [1,3,5,8]

. . . . . .

# Backtracking via Generate and Test

- In Haskell this can be implemented as follows . . .
  - Represent (partial) placement of queens as a list
  - Position `i` is column number of queen in row `i+1`
    Earlier board is represented `[1,3,5,7,2,4,6]`

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  - `allqueens n = head (drop n (iterate extendall [[]]))`

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- `extendall` maps `extend` over all placements on $k$ rows to get all placements on $k+1$ rows
- All possible placements of $n$ queens
  - `allqueens n = head (drop n (iterate extendall [[]]))`
- One placement of $n$ queens
  - `queens n = head (allqueens n)`

# Queens ...

```
n :: Int
n = ..

extend l  = [(x:l) | x <- [1..n], compat x l]

compat x []  = True
compat x l  = not (elem x l) && (notdiag x l)

extendall l = concatMap extend l

allqueens = head (drop n  (iterate extendall [[]]))
queens = head allqueens
```