Introduction to Programming: Lecture 1

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About this course ...

About this course ...

- Learn programming

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- Learn programming in Haskell

About this course ...

- Learn programming in Haskell
- Learn to think algorithmically.

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- Learn to think algorithmically.

Evaluation

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- Learn programming in Haskell
- Learn to think algorithmically.

Evaluation

- About 50% weightage to assignments.
- About 50% weightage to exams.

References

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- Lecture notes written for this course by Madhavan Mukund. (Available at `http://www.cmi.ac.in/~madhavan/courses/programming08`)

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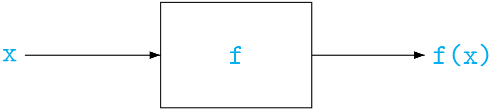
- Online book at http://learnyouahaskell.com
- Introduction to Functional Programming using Haskell by Richard Bird.
- A Gentle Introduction to Haskell by Paul Hudak et al.
- Real-world Haskell by Bryan O'Sullivan, John Goerzen and Don Stewart.

Programs as functions

Functions transform inputs to outputs:



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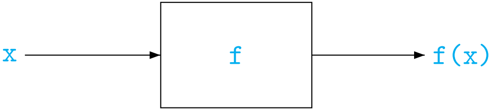

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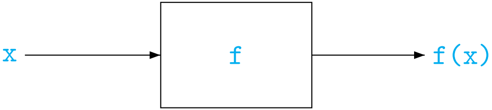

A typical program consists of rules to produce an output from an input

Computation is the process of applying the rules described by a program

Building up programs

How do we describe the rules?

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- Start with basic “built in” functions

Building up programs

How do we describe the rules?

- Start with basic "built in" functions
- Use these to build more complex functions

Building up programs . . .

Suppose

Building up programs ...

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- ...there were the whole numbers, `{0,1,2,...}`

Building up programs . . .

Suppose

- . . . there were the whole numbers, `{0,1,2,...}`
- . . . and one function, `succ` (successor)

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succ 0 = 1
succ 1 = 2
succ 2 = 3
```

. . .

Building up programs . . .

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Then, we may define `plusTwo`, as

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plusTwo n = succ (succ n)
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by composing two copies of `succ`.

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Then, we may define `plusTwo`, as

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by composing two copies of `succ`.

Composing `plusTwo` and `succ` we get

```
plusThree n = succ (plusTwo n)
```

Addition....

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- How do we describe this rule concisely for all `n` and `m`?

Recursive definitions

Goal: Define `plus n 0`, `plus n 1`, . . . , `plus n i`, . . . for each `i`

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- `plus 7 3` $=$

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  $=$ `succ (succ (plus 7 (succ 0)))`

  $=$ `succ (succ (succ (plus 7 0)))`

  $=$ `succ (succ (succ 7))` $=$ `succ (succ 8)`

  $=$ `succ 9`

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  = `succ (succ (succ (plus 7 0)))`

  = `succ (succ (succ 7))` = `succ (succ 8)`

  = `succ 9` = `10`

Recursive definitions . . .

Multiplication is repeated addition

$$\texttt{mult n m} = \underbrace{\texttt{plus n (plus n (...(plus n}}_{\texttt{m times}}\texttt{ 0)...))}$$

Recursive definitions ...

Multiplication is repeated addition

$$\texttt{mult n m} = \underbrace{\texttt{plus n (plus n (...(plus n}}_{\texttt{m times}}\texttt{ 0)...))}$$

The rule for multiplication

- `mult n 0 = 0`, for all `n`
- `mult n (succ m) = plus n (mult n m)`, for all `n` and `m`

Types

Functions operate on values of a fixed type

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Other types

- `capitalize 'a' = 'A'`, `capitalize 'b' = 'B'`, ...
- Inputs and outputs are letters or "characters"

Functional programming

Haskell: a programming language for describing functions

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A description in Haskell of a function `f` has two parts:

1. Types of inputs and outputs
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Example:

`sqr :: Int -> Int`	Type definition
`sqr x = x*x`	Computation rule

Basic types and operations in Haskell

- `Int` Integers
  - Operations `+`, `-`, `*`
  - Functions `div`, `mod`
  - Note: `/` takes two `Int`s as input and produces a `Float`
- `Float`
- `Char`
  - Values written in single quotes — `'z'`, `'&'`, ...
- `Bool`
  - Values `True` and `False`.
  - Operations `&&`, `||`, `not`

Defining functions

- Boolean expressions
  - Comparisons on `Int`: `==`, `/=`, `<`, `<=`, `>`, `>=`
  - Boolean combinations `&&` (and), `||` (or) and `not` (negation).

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```
inorder:: Int -> Int -> Int -> Bool
inorder x y z = (x <= y) && (y <= z)
```

Definition by cases: Pattern matching

- Defining by pattern matching

```
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xor True  False = True
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- Use first definition that matches, top to bottom
- `xor False True` matches second definition
- `xor True True` matches third definition

Definition by cases: Pattern matching

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```
or :: Bool -> Bool -> Bool
or True b  = True
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- `or True False` matches first definition
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Definition by cases: Pattern matching

```
and :: Bool -> Bool -> Bool
and  True b  = b
and  False b = False
```

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- As we saw earlier, many functions are defined recursively
  - Base case: Explicit value for $f(0)$
  - Inductive step: Define $f(n)$ in terms of n and $f(n-1), \ldots, f(0)$

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- Note the bracketing in `factorial (n-1)`
  - `factorial n-1` would be bracketed as `(factorial n) -1`
- No guarantee of termination!
  - What does `factorial (-1)` generate?

Functions with multiple inputs

```
plus m n = m + n
```

- What is the type of `plus`?
  - Mathematically, $\mathit{plus} : \mathbb{Z} \times \mathbb{Z} \rightarrow \mathbb{Z}$
- Need to know arity of functions

Functions with multiple inputs ...

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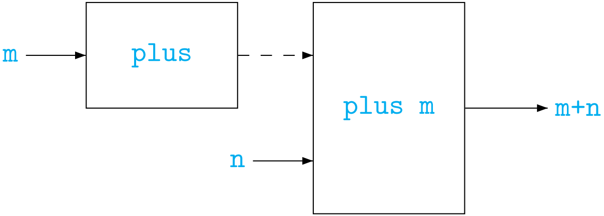

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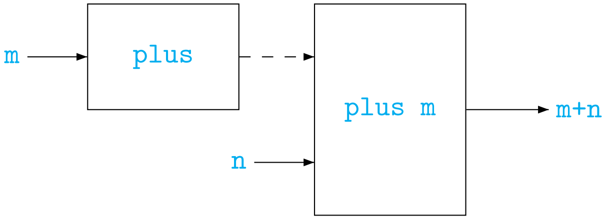

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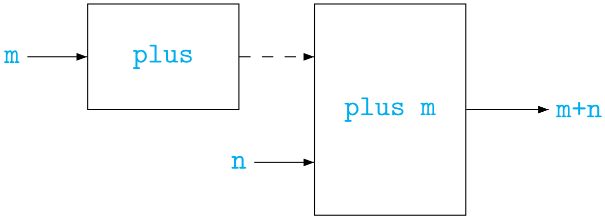

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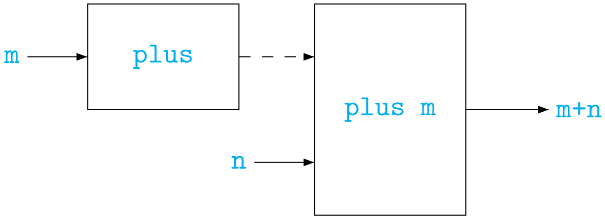

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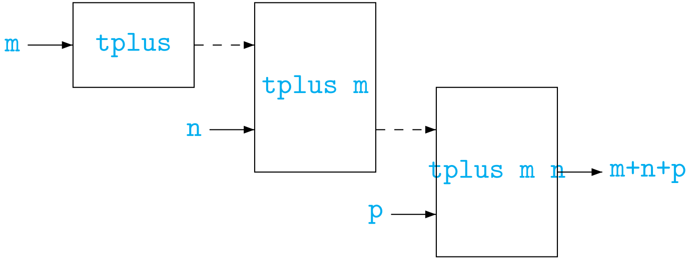
  - `plus :: Int -> (Int -> Int)`

Functions with multiple inputs . . .

- `tplus m n p = m + n + p`



- `tplus m n p :: Int -> (Int -> (Int -> Int))`

Running Haskell programs

- Write your Haskell program using a text editor
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- Experiment with `ghci` as a "calculator"

Conditional definitions

- Conditional definitions using guards
- For instance, “fix” the function to work for negative inputs

```
factorial :: Int -> Int
factorial 0 = 1
factorial n
  | n < 0 = factorial (-n)
  | n > 0 = n * (factorial (n-1))
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- Second definition has two parts
  - Each part is guarded by a condition
  - Guards are tested top to bottom
- Indentation to show that definition continues on multiple lines
- Multiple definitions could have different forms
  - Pattern matching for `factorial 0`
  - Conditional definition for `factorial n`

Conditional definitions . . .

- Guards may overlap

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- Guards may not cover all cases

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  | n > 0 = n * (factorial (n-1))
```

- Guards may not cover all cases

```
factorial :: Int -> Int
factorial 0 = 1
factorial n
  | n < 0 = factorial (-n)
  | n > 1 = n * (factorial (n-1))
```

- No match for `factorial 1`

```
Program error: pattern match failure: factorial 1
```