

Introduction to Programming: Lecture 5

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Mark Lists

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- ▶ Rearrange it so that you have a list of lists in which each list gives the marks obtained by one student.

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transpose [[10,10,8], [9,2,10]]  
          = [[10,9],[10,2],[8,10]]
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```
transpose []:xs = []  
transpose xs = (map head xs):transpose (map tail xs)
```

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► Arithmetic progressions

$[1, 3..8] = [1, 3, 5, 7]$

$[2, 5..19] = [2, 5, 8, 11, 14, 17]$

► Lists in descending order

$[8, 7..5] = [8, 7, 6, 5]$

$[12, 8..-9] = [12, 8, 4, 0, -4, -8]$

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- ▶ `([Int],Int)` is the type whose members are pairs, the first is a list of integers and the second is an integer.

`([],67) :: ([Int], Int)`

`([1,2],73) :: ([Int],Int)`

Tuples: `fst` and `snd`

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- ▶ `fst([1,2,3], 'a') = [1,2,3]`
- ▶ `snd ([1,2,3], 'a') = 'a'`

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- ▶ `sumpairlist l = sum ((map sumpairs) l)`

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- ▶ A tuple of type `(String,Int)` can store a student's mark and a marklist is of type `[(String,Int)]`.

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lookUp :: String -> [(String,Int)] -> Int  
lookUp s ((name,marks):l)  
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getStudMarks :: String -> [[(String,Int)]] -> [Int]  
getStudMarks s = map (lookUp s)
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- ▶ We may then write

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lookUp :: String -> Marklist -> Int
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getStudMarks :: String -> [Marklist] -> [Int]
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and so on.

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The function `zip`

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 - ▶ `zip ['a','b','c'] [1..3] ~> [('a',1), ('b',2), ('c',3)]`
 - ▶ `zip ['a'..'z'] [1..10] ~> [('a',1), ('b',2), ..., ('j',10)]`

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totalMarks [("Amitabh", 80), ("Smita", 90)],  
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```
totalMarks (x:xs) =  
  zip (map fst x) (addMarks (map (map snd) (x:xs)))
```

Idiomatic programming

- ▶ Programming languages are ... **languages**!
- ▶ Like “natural languages”, we can say the same thing in many ways
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- ▶ To learn a language, you must practice speaking it.

Example: initial segments

- ▶ Write a Haskell function `initsegs` which returns the list of initial segments of a list.

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initsegs [1,2,3] = [], [1], [1,2], [1,2,3]  
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initsegs (x:xs) = [] : map (x:) (initsegs xs)
```

Example: interleave

- `interleave x l` inserts `x` into all possible positions in the list `l`

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interleave 3 [] = [[3]]
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interleave 3 [2,3] = [[3,2,3],[2,3,3],[2,3,3]]
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interleave 'a' "abcd" =  
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```
interleave x [] = [[x]]
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```
interleave x (y:ys) = (x:y:ys) :  
    map (y:) (interleave x ys)
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Exercise: What is the type of `concatMap`

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```
part [x] = [[[x]]]  
part (x:xs) = map ([x]:) (part xs) ++  
              map (f x) (part xs)  
where  
  f x (y:ys) = (x:y):ys
```