Introduction to Programming: Lecture 14

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Arrays

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  The module `Data.Array` has to be imported to use arrays.

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- `listArray` takes a range of values from the index type and list of values to create an array.
- Accessing an element is done using the `!` operator:

```
myarray!1 = "two"
```

Arrays ...

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More about indices

- Any type `a` that belongs to the typeclass `Ix` must provide the following functions:

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range :: (a,a) -> [a]
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- `inRange` checks of the value lies within the range.

```
inRange (0,5) 6 = False
```

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- We may also create an array by supplying the index-value pairs (associative list) as a list.

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Searching in a sorted array

Searching for 77

5 13 16 22 27 33 41 55 61 70 77 83 85 91 93 99

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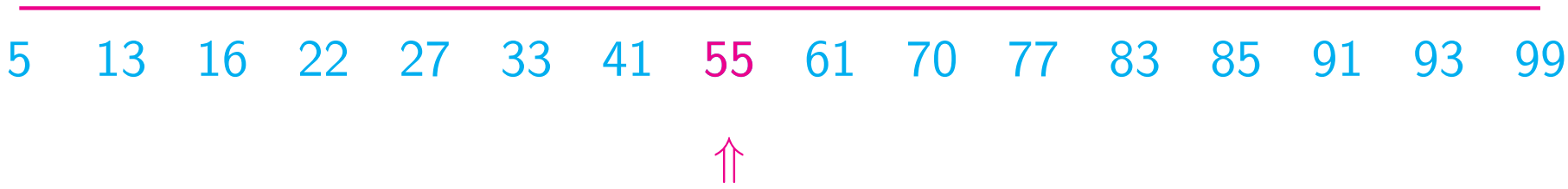

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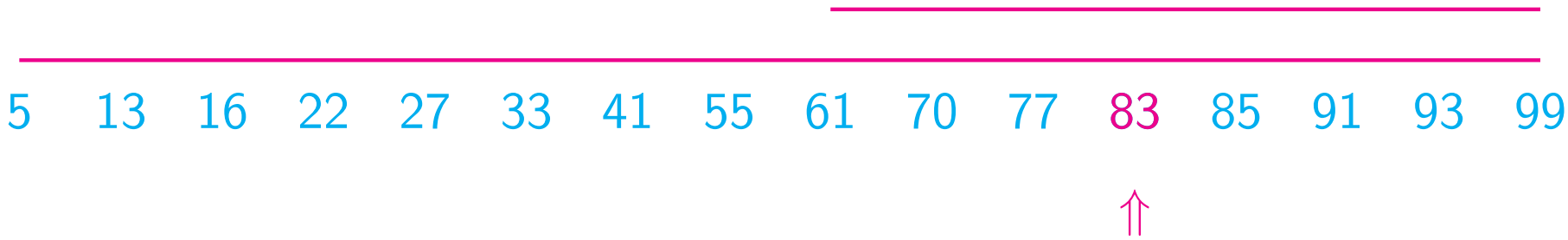

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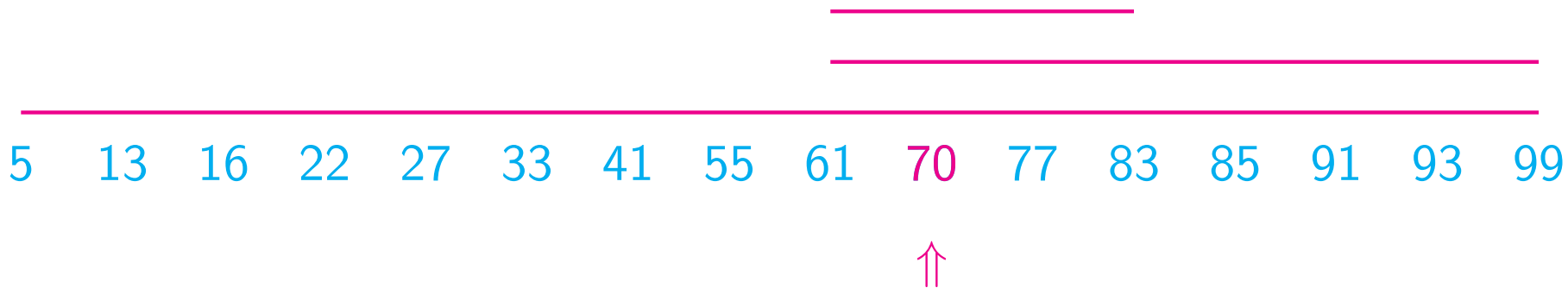

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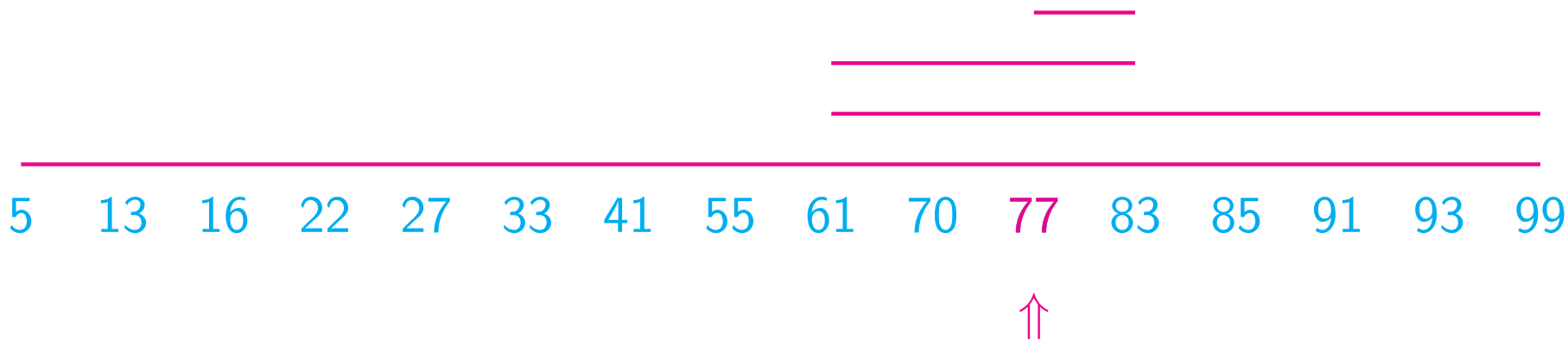

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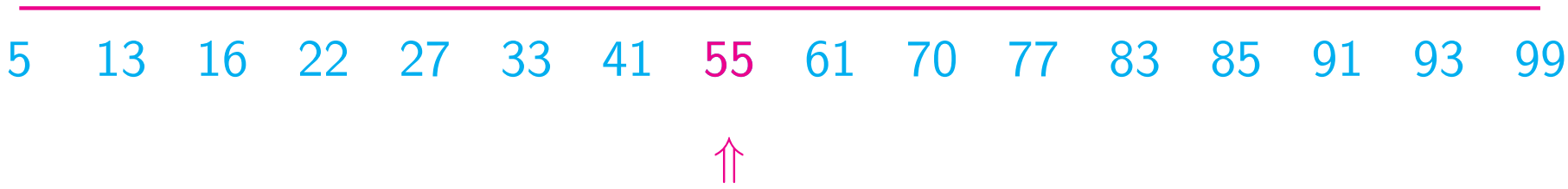

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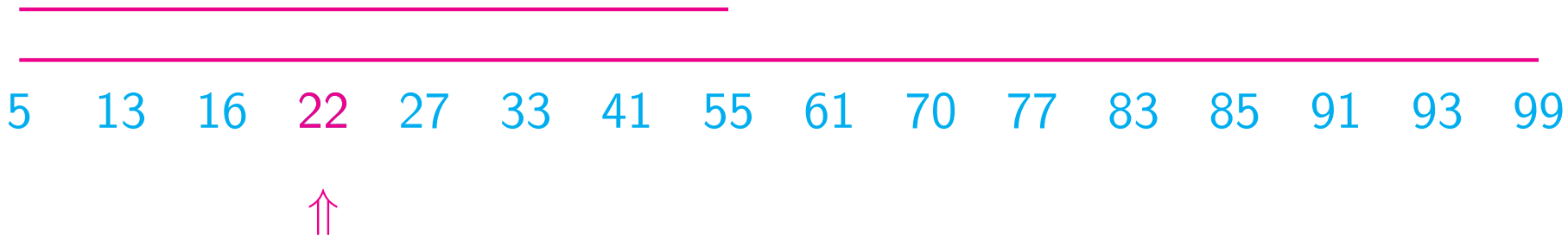

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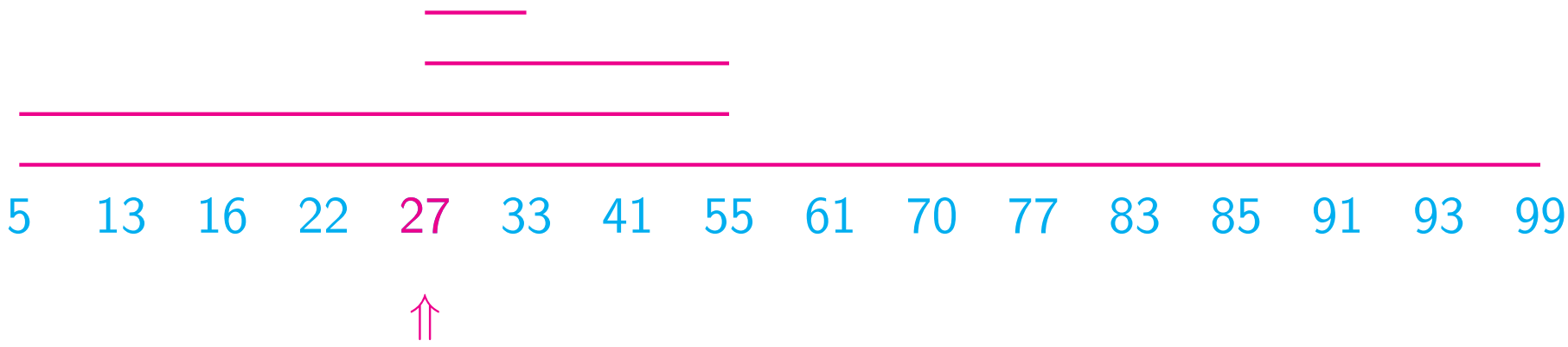

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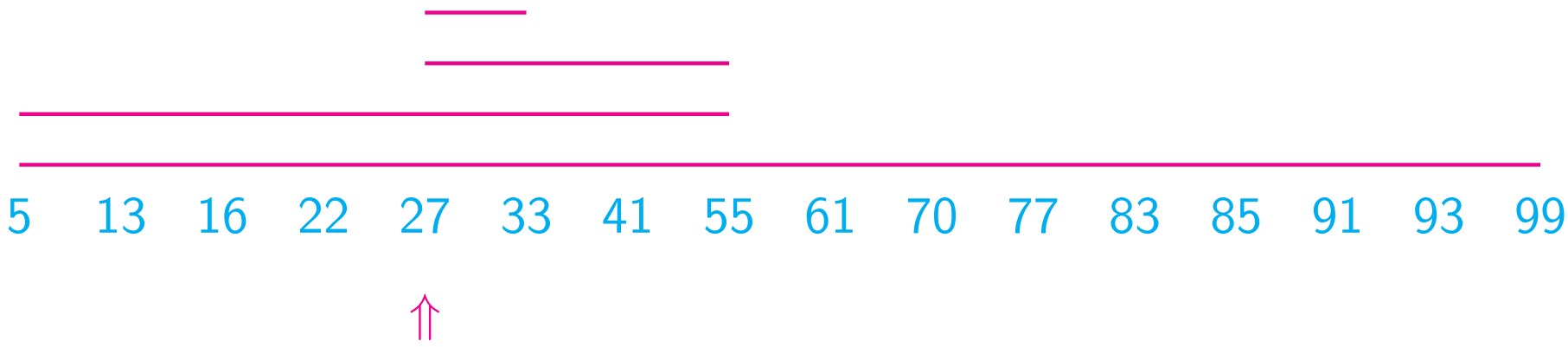

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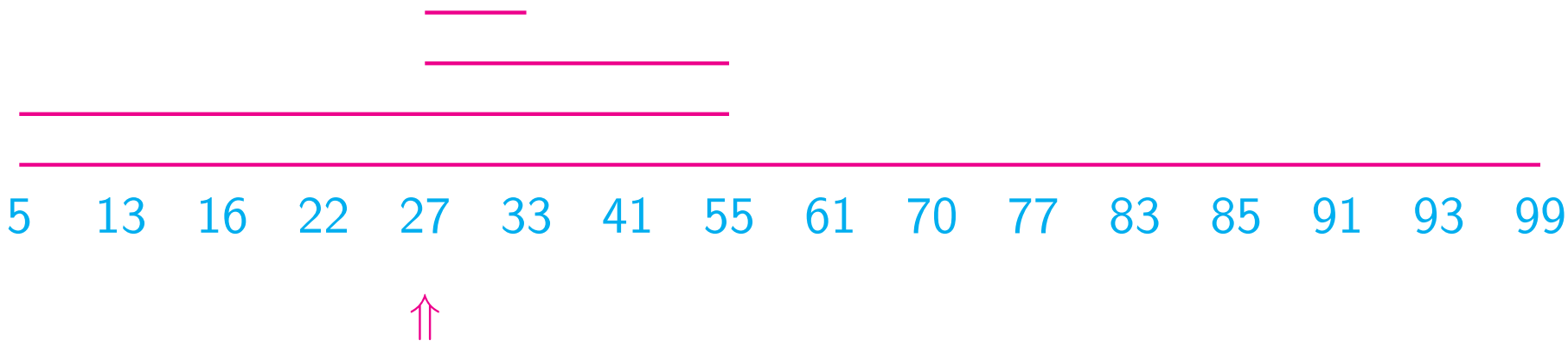

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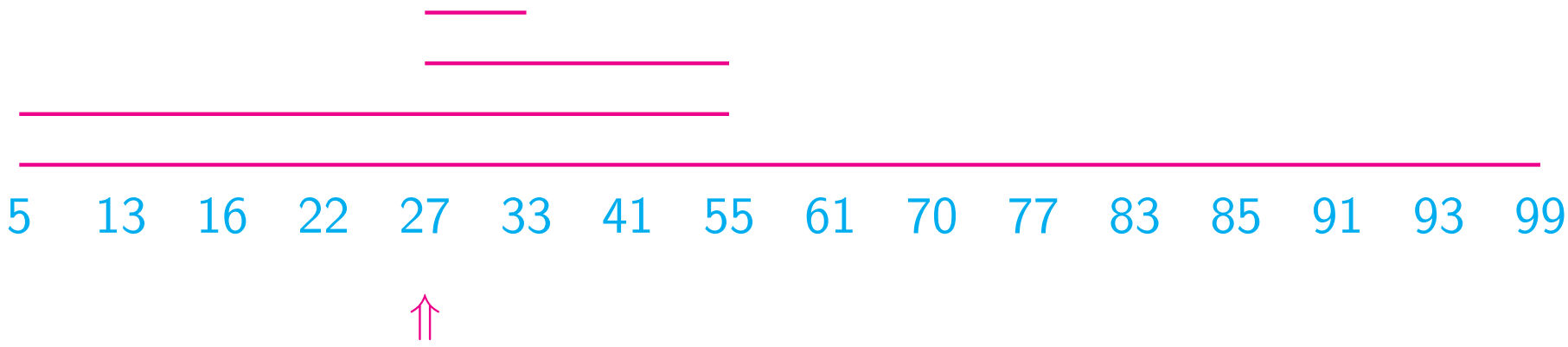

- Each step halves the interval to search
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- Searching a sorted list of size N takes $\log_2 N$ steps

Binary Searching in Haskell

- The following function checks if the value `v` appears between positions `b` and `e` in the array `ar`

```
import Data.Array
bAux :: Ord a => (Int,Int) -> a ->
                              Array Int a ->  Bool
bAux (b,e) v ar
  | b > e       = False
  | (ar!m) == v = True
  | (v < ar!m)  = bAux (b,m-1) v ar
  | otherwise   = bAux (m+1,e) v ar
  where
   m = div (b+e) 2
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- Searching for a value `v` in a sorted array `ar`

```
bsearch :: Ord a => a -> Array Int a -> Bool
bsearch v ar = bAux (bounds ar) v ar
```

Arrays to Lists and Back

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- To convert a list into an `Int` indexed array, we can use

```
listToArray ls = listArray (0,l-1) ls
  where
  l = length ls
```

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- Use `bsearch` to carry out the searches efficiently.

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- `assocs` returns the associative list describing the array.

  ```
  assocs myarray =
  [(0,"one"),(1,"two"),(2,"three")]
  ```

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- However, note that updating an array using the `//` operator is an expensive operation with cost proportional to the size of the array.
- Efficiently updatable arrays can be created in Haskell, but that needs additional concepts.

Another array constructing function

- The `accumArray` function takes a "accumulating" function and an associative list and creates an array.

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accumArray (+) 100 (0,2) [(0,1),(1,3),(2,4)]
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- Also works in linear time on the length of the associative list plus the range.

An old example: `minout`

- `minout :: [Int] -> Int`
  `minout l` is the minimum nonnegative number not in `l` assuming that all elements in `l` are nonnegative and distinct.
  - `minout [3,1,2] = 0`
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- How do we compute `minout`?
- The linear time solution via lists involved a rather clever divide and conquer algorithm.
- With arrays the solution is simpler

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    `[(i,1) | i <- ls, 0 <= i, i <= m]`
  - Accumulate values from this associative list using the function
    `f x y = y`
  - The index of the first entry in the array with `0` is the answer.

minout via arrays ...

```
import Data.Array

minout ls = firstZero 0
  where

  m = length ls
  f x y = y

  myArray = accumArray f 0 (0,m)
      [(i,1) | i <- ls, 0 <= i, i <= m]

  firstZero :: Int -> Int
  firstZero i
   | (myArray!i == 0) = i
   | otherwise = firstZero  (i+1)
```

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- So, two or k-dimensional arrays are essentially same, with just a different sent of indices.

Two dimensional arrays

- The definition of an array makes no reference to a dimension.
- So, two or k-dimensional arrays are essentially same, with just a different sent of indices.
- Here is way to generate an $n \times n$ identity matrix.

```
idMat n = accumArray f 0 ((0,0),(n-1,n-1))
            [((i,i),1) | i <- [0..(n-1)]]
  where
  f x y = y
```