Introduction to Programming: Lecture 6

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27 August 2013

Folding from the Left

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- `foldl ::  (a -> b -> a) -> a -> [b] -> a`

  ```
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  foldl f v (x:xs) = foldl f (f v x) xs
  ```

foldl

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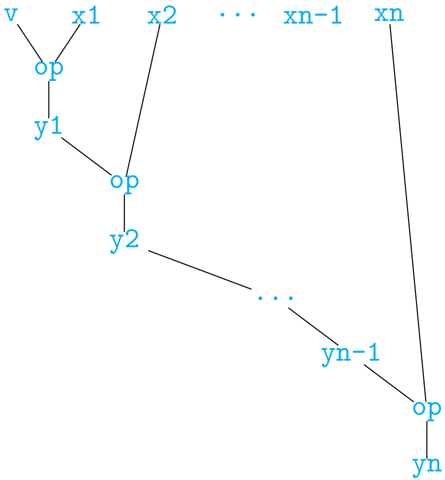

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nextdigit :: Int -> Char -> Int
nextdigit i c = 10*i + (chartonum c)
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- What next?

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  strtonum = foldl nextdigit 0
  ```

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  `map == map` will result in a type error. There is no equality defined on the type of `map`
- Order (`<`) is also not defined on higher order types.

  In general it is not possible to check if two functions compute the same values on all inputs.

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[1,2,3] = [1,2,3]
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```

- Two tuples are equal if and only if the corresponding positions have equal values.

```
(True, False) = (True,False)
(True, 17, False) = (True, 17, False)
(True, 17, False) /= (False, 17, True)
```

Order in lists and tuples

- Tuples of the same type are ordered lexicographically provided all the underlying types are ordered.

```
(False, 17, 24) < (False, 18, 1)
(False, 89) < (False, 100)
```

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  `(89,(+)) < (90,(+))` will result in a type error.

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  ```
  [1,2,3] < [1,2,3,0]
  [] < [1]
  [1,2,3] < [1,2,4]
  ```

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  insert 3 [] = [3]
  ```

- ```
  insert Int -> [Int] -> [Int]
  insert x [] = [x]
  insert x (y:ys)
    | (x <= y) = x:y:ys
    | otherwise = y : insert x ys
  ```
  ```

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- Note that `insert` is not polymorphic, in the sense that we have seen so far, as `<=` is not necessarily defined on all types.
  ```

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or more succinctly `isort = foldr insert []`

Sorting: Merge Sorting

- Here is another method to sort a list.
  - Divide the list into two halves.
  - Sort each recursively using this algorithm.
  - Merge together the two sorted lists into a single sorted list.

Merging two sorted lists

5	16	83	99	33	55	85	93

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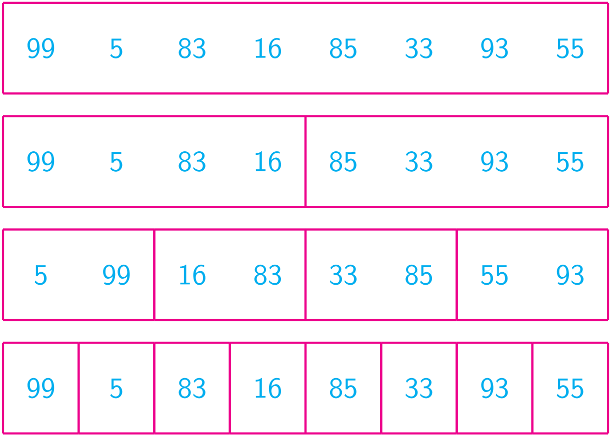

99 5 83 16 85 33 93 55
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merge :: [Int] -> [Int] -> [Int]
merge [] l = l
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   | (x < y) = x : merge xs (y:ys)
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Mergesort in Haskell

- Using `merge` to write down a `mergesort`

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```
mergesort :: [Int] -> [Int]
mergesort [] = []
mergesort [x] = [x]
mergesort l = merge (mergesort fhalf)
                    (mergesort shalf)
              where
              fhalf = take n l
              shalf = drop n l
              n = div (length l) 2
```

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  Median is not easy to find.

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  - L_1 is the list of elements of L that are smaller than p and
  - L_2 is the list of elements of L that are at least as big as p.
- Sort L_1 and L_2 recursively.

Quicksort ...

265 319 389 345 159 267 348 365 128

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265	319	389	345	159	267	348	365	128
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Quicksort ...

```
quicksort :: [Int] -> [Int]
quicksort [] = []
quicksort (x:xs) = (quicksort lower) ++
                   [splitter] ++
                   (quicksort upper)
  where
    splitter = x
    lower    = filter (<= x) xs
    upper    = filter (> x) xs
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- In the worst case, `lower` or `upper` is empty
  - For our choice of splitter, worst case input is any sorted list

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```
[1,2,3] ++ [4,5,6] ->
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1:(2:(3:([] ++ [4,5,6])))  ->
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```

- In `l1 ++ l2` we use the second rule `length l1` times and the first rule once
- It takes as many steps as `length l1 + 1`

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  $T(n) = n$ for ++ where n is the length of the left argument.

- The function ++ takes the same number of steps on inputs of the same length.

  This need not always be the case.

Efficiency ...

```
elem :: Int -> [Int] -> Bool
elem i [] = False
elem i (x:xs)
  | (i==x) = True
  | otherwise = elem i xs
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  elem 3 [7,8,9] ->
    elem 3 [8,9] ->
      elem 3 [9] ->
        elem 3 [] ->
          False
```

- What do we take the value of $T(n)$ to be?

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- Worst-case is pessimistic. But it assures us that on any input of length `n` the program does NOT take more than `T(n)` steps.
- Average-case is perhaps a more accurate description. But is usually very difficult to calculate.

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  - The average among all inputs of length `n`. Average-case complexity
- Best-case complexity is useless. For eg. it suggests that `elem` returns the answer in one step on any inputs of any length `n`.
- Worst-case is pessimistic. But it assures us that on any input of length `n` the program does NOT take more than `T(n)` steps.
- Average-case is perhaps a more accurate description. But is usually very difficult to calculate.
- For the purposes of this course, we stick to worst-case analysis.

Measuring efficiency in Haskell ...

- What is the complexity of `reverse`?

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reverse [] = []
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- Expand and solve

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\begin{aligned}
T(n) &= T(n-1) + n \\
&= (T(n-2) + n-1) + n \\
&= (T(n-3) + n-2) + n-1 + n \\
&= \cdots \\
&= T(0) + 1 + 2 + \cdots + n \\
&= 1 + 1 + 2 + \cdots + n \\
&= n(n+1)/2 + 1
\end{aligned}
$$

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- Clearly `transfer l1 l2 = (reverse l1)++l2`
- Therefore `reverse l = transfer l []`
- For `transfer`, input size is `length l1`

$$\begin{aligned} T(0) &= 1 \\ T(n) &= T(n-1) + 1 \end{aligned}$$

- Thus $T(n) = n + 1$